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Partially Cooperating Duopoly Games: Substitute and Complementary Goods

Kazuhiro Ohnishi
Institute for Economic Sciences, 1-3-10 Nishimaiko Tarumi, Kobe 655-0048, Japan
E-mail: ohnishi@e.people.or.jp

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Abstract: This paper examines a two-stage Cournot duopoly model in which two firms coexist with each other. In the first stage, each firm simultaneously and independently decides the degree of common ownership. In the second stage, after observing the rival's decision in the first stage, each firm simultaneously and independently chooses its output level. There is no possibility of entry or exit. This paper considers both cases of substitute and complementary goods. The paper shows that the degrees of common ownership are positive at the equilibrium solutions of the two duopoly cases. Furthermore, the paper compares the equilibrium outcomes of the two duopoly cases, and shows that the equilibrium profits of the complementary goods duopoly game are higher than those of the substitute goods duopoly game. As a result of this analysis, the paper finds that partial cooperation between the firms is more profitable in the complementary goods duopoly game than in the substitute goods duopoly game.

Keywords: Complementary goods, partially cooperating firms, quantity competition, substitute goods

JEL classification: C72; D21

1. INTRODUCTION

The seminal paper by Cyert and DeGroot (1973) introduced the concept of "partial cooperation", where each firm maximizes the sum of its own profit and certain proportions of the profits of its competitors. Since then, the theoretical analysis of oligopoly markets that incorporate partially cooperating firms has been widely performed by many researchers (e.g., see Chen, Matsumura and Zeng, 2021; Cracau, 2015; Escrihuela-Villar, 2015; Hirose and Matsumura, 2022; López and Vives, 2019; Matsumoto, Merlone and Szidarovszky, 2010; Sato and Matsumura, 2020; Szidarovszky, 2008). For example, Escrihuela-Villar (2015) discusses the relationship between

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two oligopoly models: the conjectural variations approach (Bowley, 1924) and the coefficient of cooperation (Cyert and DeGroot, 1973), and shows that in a general symmetric quantity-setting oligopoly, the conjectural variations solution replicates that of a model where the coefficient of cooperation is interpreted as a measure of the degree of strategic interaction among firms. Szidarovszky (2008) examines Cournot oligopoly games in which firms might face capacity limits, thresholds for minimal and maximal moves, and antitrust thresholds in the case of partial cooperation, and shows that there are cases without equilibrium and also cases with multiple, sometimes infinity many equilibrium solutions. Matsumoto, Merlone and Szidarovszky (2010) introduce a general framework of partial cooperation and shareholding interlocks in Cournot oligopoly models, and examines the dependence of the equilibrium on model parameters and the asymptotic properties of the dynamic extensions under discrete time scales. The authors provide conditions for the local asymptotical stability of the equilibrium solutions requiring that the speeds of adjustments of firms be sufficiently small. In addition, Hirose and Matsumura (2022) examine how common ownership affects firms' voluntary commitment with emission restrictions and emissions abatement activities in an oligopoly, and show that an increase in the degree of common ownership may reduce emissions abatement activities unless the degree of common ownership is small.

In this paper, we investigate a two-stage quantity-setting duopoly model in which two firms coexist with each other. At stage one, each firm simultaneously and independently decides the degree of common ownership. At stage two, after observing the rival's decision at stage one, each firm simultaneously and independently chooses its output level. There is no possibility of entry or exit. We consider both cases of substitute and complementary goods. We present the respective equilibrium outcomes of the two duopoly cases. In addition, we compare the two duopoly cases.

The structure of this paper is as follows. In Section 2, we introduce the basic setting. Section 3 solves and compares the two duopoly cases. Finally, Section 4 concludes the paper.

2. BASIC SETTING

Consider a market where there are two firms: firm 1 and firm 2. In the remainder of this paper, subscripts 1 and 2 denote firm 1 and firm 2, respectively. Furthermore, when i and j are used to refer firms in an expression, they should be understood to represent to 1 and 2 with $i \neq j$. There is a continuum of consumers of the same type, and the representative consumer maximizes consumer surplus:

$$CS = U(q_1, q_2) - p_1 q_1 - p_2 q_2, \tag{1}$$

where q_i represents the amount of good i and p_i is its price. The function $U(q_1, q_2)$ is quadratic and symmetric in q_1 and q_2: $U(q_1,q_2) = (q_1+q_2) - (q_1^2+q_2^2-q_1q_2)/2$. The inverse demand (price) function is given by $p_i = 1 - q_i - \delta q_j$, where δ represents the degree of product differentiation. If $\delta \in (0, 1)$, goods are substitutive, and if $\delta \in (-1, 0)$, goods are complementary, Therefore, firm i's profit function is given by

$$\pi_i = q_i(1 - q_i + \delta q_j). \tag{2}$$

Firm i's objective function V_i is given by

$$V_i = \pi_i + \theta_i.\pi_j, \tag{3}$$

where $\theta \in (0, 1)$ θ represents the degree of common ownership.

The timing of the game is as follows. In the first stage, each firm i independently chooses θ_i. In the second stage, after observing the rival's choice in the first stage, each firm i independently chooses its output level q_i. Our equilibrium concept is subgame perfection and the two cases of the next section are solved by backward induction.

3. RESULTS

In this section, we examine the following two cases: duopoly competition with substitute goods and duopoly competition with complementary goods.

3.1. Substitute Goods

If $\theta \in (0,1)$, it is a measure of the degree of substitutability among products. For the sake of simplicity, we assume $\delta = 0.5$.

We solve the game by backward induction. In the second stage, firm i chooses its output q_i in order to maximize its objective function (3). By differentiating (3) with respect to q_i, we obtain firm i's best reaction function in quantity:

$$q_i(q_j) = \frac{2 - (1+\theta_i)\, q_j}{4}. \tag{4}$$

Furthermore, from (4), we can derive the Cournot equilibrium quantity in terms of θ_i and θ_j:

$$q_i = \frac{2(3-\theta_i)}{15 - \theta_i - \theta_j - \theta_i\theta_j}. \tag{5}$$

In the first stage, each firm i anticipates these quantities and chooses θ_i in order to maximize the corresponding profit:

$$\pi_i = \frac{2\left(18 - 3\theta_i - \theta_i^2 - 3\theta_i\theta_j - \theta_i^2\theta_j\right)}{225 - 30\theta_i - 30\theta_j + \theta_i^2 + \theta_j^2 - 28\theta_i\theta_j + 2\theta_i^2\theta_j + 2\theta_i\theta_j^2 + \theta_i^2\theta_j^2}. \tag{6}$$

By differentiating (6) with respect to θ_i, we obtain the best response:

$$\theta_i(\theta_j) = \frac{36}{(\theta_j - 15)^2}. \tag{7}$$

Using the symmetry of firms, we obtain the equilibrium weight $\theta_i = \theta^S \approx 0.164$. Furthermore, we can obtain the corresponding quantity $q_i = q^S \fallingdotseq 0.3873$ and profit $\pi_i = \pi^S \fallingdotseq 0.1623$.

3.2. Complementary Goods

If $\theta \in (-1,0)$, it represents the degree of complementarity among products. For simplicity, we assume $\delta = -0.5$.

At stage two, firm *i* chooses its output q_i in order to maximize its objective function (3). By differentiating (3) with respect to q_i, we obtain firm *i*'s best reaction function in quantity:

$$q_i(q_j) = \frac{2 + (1 + \theta_i)q_j}{4}, \tag{8}$$

Furthermore, from (8), we can derive the Cournot equilibrium quantity in terms of θ_i and θ_j:

$$q_i = \frac{2(5 + \theta_i)}{15 - \theta_i - \theta_j - \theta_i\theta_j}, \tag{9}$$

At stage one, each firm *i* anticipates these quantities and chooses θ_i in order to maximize the corresponding profit:

$$\pi_i = \frac{2(50 - 5\theta_i - 3\theta_i^2 - 5\theta_i\theta_j - \theta_i^2\theta_j)}{225 - 30\theta_i - 30\theta_j + \theta_i^2 + \theta_j^2 - 28\theta_i\theta_j + 2\theta_i^2\theta_j + 2\theta_i\theta_j^2 + \theta_i^2\theta_j^2}, \tag{10}$$

By differentiating (10) with respect to θ_i, we have the best response:

$$\theta_i(\theta_j) = \frac{100}{(\theta_j - 15)^2}, \tag{11}$$

Using the symmetry of firms, we have the equilibrium weight $\theta_i = \theta^C \approx 0.474$. Furthermore, we obtain the corresponding quantity $\pi_i = \pi^C \approx 0.4783$ and profit $\pi_i = \pi^C \approx 0.4783$.

Finally, we compare the two economic regimes (substitute goods duopoly and complementary goods duopoly). From the above results, we present the following proposition.

Proposition 1: In the two Cournot duopoly games, $\theta^S < \theta^C$, $q^S < q^C$, and $\pi^S < \theta^C$.

This proposition states that partially cooperation between the firms is more profitable in the complementary goods duopoly game than in the substitute goods duopoly game.

4. CONCLUSION

We have investigated two-stage Cournot duopoly competition in which two firms coexist with each other. We have considered both games of substitute and complementary goods. It has been shown that the degrees of common ownership are positive at the equilibrium outcomes of the two duopoly games. We have examined two-stage games. However, in the real world, firms are generally faced with long-run competition. Therefore, in the near future, we will examine various dynamic models consisting of partially cooperating firms.

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